University of Bahrain
College of Information Technology
Department of Computer Science

ITCS252 Discrete Mathematics

Second Semester 2011/2012

Second Midterm Exam — 60 minutes.

Time: 2:00 – 3:00 PM

STUDENT NAME	
STUDENT#	
SECTION	4

QUESTION#	MARKS		COMMENTS
1	10	7	
2	8	8	
3	6	5 +0.5	Dr. Ali Alsaffar
4	8	8	
5	8	8	
TOTAL	40		

Instructors: Dr. Ali Alsaffar.
Dr. Ali Khan (Coordinator).
Dr. Nabil Hewahi.
Dr. Youssef Harrath.

Answer All Questions

Q1. Let $S(x)$: "x is a student."
$M(x)$: "x is smart."
$I(x)$: "x is intelligent."

(a) **[2 pts.]** Simplify the following so that your answer does not have the negation symbol ($\neg$).

$$\neg\exists x[(\neg I(x) \vee \neg M(x)) \wedge \neg S(x)].$$

$\neg \exists x[(\neg I(x) \vee \neg M(x)) \wedge \neg S(x)]$

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$\forall x \; \neg(\neg I(x) \vee \neg M(x)) \vee \neg\neg S(x)$

$\forall x \; (I(x) \wedge M(x)) \vee S(x)$

(b) **[2 pts.]** Write in words the simplified statement.

2 All people are intelligent and smart or they are student

(c) Write the following statements in symbolic forms. Assume the domain is the set of all UOB students N.

(1) **[3 pts.]** No student is smart and intelligent. $\forall x \in N : \neg(M(x) \wedge I(x))$

1.5 $\forall x \in N: \cancel{S(x) \rightarrow} (M(x) \wedge I(x))$

(2) **[3 pts.]** Not all students are smart. $\exists x \in N: \neg M(x)$

1.5 $\exists x \in N: \neg(S(x) \rightarrow M(x))$

Q2. **[8 points]** For sets A, B, and C, prove that $(A - C) \cap (C - B) = \phi$

L.H.S: $(A-C) \cap (C-B)$

$(A \cap \overline{C}) \cap (C \cap \overline{B})$

$A \cap \overline{C} \cap C \cap \overline{B}$

8 $A \cap \emptyset \cap \overline{B} = \emptyset = \text{R.H.S}$

Q3. For each of the following propositions, indicate whether it is true or false. ***Justify your answer.***

(a) [**2 pts.**] ☐ True ☑ False There exists a prime number p such that $2^p = 125$. prime = {2, 3, 5, 7 ...}

1.5

For $p = 5 \to 2^5 = 32$, For $p = 7 \to 2^7 = 128$

$\therefore 2^5 < 2^p < 2^7$ $p = 2, 3, \ldots$

$\therefore 32 < 125 < 128$ $\therefore$ False

(b) [**2 pts.**] ☑ True ☐ False $\exists x \geq 0$ such that $x^2 - 9 = 0$.

$1\frac{1}{2}$

$\boxed{\text{For } x = 3 \quad x^2 - 9 = 0 \to (3)^2 - 9 = 0}$ ✓

✗ For $x = 4$ $(4^2) - 9 = 7$ $\therefore$ true

المفترض اعطي مثال واحد فقط !!

(c) [**2 pts.**] ☐ True ☑ False For every real numbers x and y, if $x^2 - y^2 = 0$, then $x = y$.

2

For $x = 1$ and $y = -1$

$x^2 = 1$ and $y^2 = 1$ $\therefore x^2 - y^2 = 0$

but $x \neq y$ $x = 1$ and $y = -1$ $\therefore$ False

Q4. [**8 pts.**] Prove that for all integers n, if n is not divisible by 3, then $n^2 - 1$ is divisible by 3.
Hint: Study the cases of the possible remainders of the division of n by 3.

Hypothesis: $n \neq 3K$, $n, K \in \mathbb{Z}$

Conclusion: $n^2 - 1 = 3K$, Direct proof

case(1): Assume $n = 3K + 1$ not div. by 3

$\therefore n^2 - 1 = (3K + 1)^2 - 1$

$= 9K^2 + 6K + 1 - 1$

$= 3(3K^2 + 2) = 3K_1$, $K_1 \in \mathbb{Z}$ $\therefore$ div. by 3

case(2): Assume $n = 3K + 2$ not div. by 3

$\therefore n^2 - 1 = (3K + 2)^2 - 1$

$= 9K^2 + 12K + 4 - 1$

8

$= 3(3K^2 + 4K + 1) = 3K_2$, $K_2 \in \mathbb{Z}$ $\therefore$ div. by 3

$\therefore n^2 - 1$ is div. by 3

Q5. [**8 points**] Given the following sequence formula:

$U_1 = 2,$

$U_{n+1} = U_n + 3$ for all integers $n \geq 1$.

$n=1 \to U_2 = U_1 + 3 \to U_2 = 2 + 3 = 5$

$n=2 \to U_3 = U_2 + 3 \to U_3 = 5 + 3 = 8$

$n=3 \to U_4 = U_3 + 3 \to U_4 = 8 + 3 = 11$

$n=4 \to U_5 = U_4 + 3 \to U_5 = 11 + 3 = 14$

(a) Find the following: 2

$U_2 =$ 5, $U_3 =$ 8, $U_4 =$ 11, $U_5 =$ 14

(b) Prove by induction that $U_n = 3n - 1$, for all $n \geq 1$.

let P(n) be a bove formela

Induction Proof :-

Basis : n = 1

$P(1) : U_1 = 3(1) - 1 = 2$ 1½ ∴ true

Assumption : n = K

$P(K) : U_K = 3K - 1$ IH 1½

Induction : n = K+1 1½

$P(K+1) : U_{K+1} = 3(K+1) - 1$

$L.H.S = U_{K+1}$

$= \underbrace{U_K}_{IH} + 3$

8

1½

$= 3K - 1 + 3$

$= 3(K+1) - 1 = R.H.S$

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